**Novel hybrid machine learning models for failure mode identification and shear strength prediction of rectangular hollow RC columns subjected to compressive and lateral loads**

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**1. Results of shear strength prediction**

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**Fig. S-1.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 50.

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**Fig. S-2.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 100.

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**Fig. S-3.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 150.

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**Fig. S-4.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 200.

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**Fig. S-5.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 250.

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**Fig. S-6.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 300.

**Table S-1** Performance of MFO-GB models on the training set

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Pop** | **Tr.atio** | **R2** | **Score** | **A10** | **Score** | **RMSE** | **Score** | **MAE** | **Score** | **Total** |
| 50 | 0.55 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.60 | 1 | 4 | 1 | 8 | 3.344 | 12 | 2.278 | 11 | 35 |
|  | 0.65 | 1 | 4 | 1 | 8 | 0.179 | 18 | 0.11 | 18 | 48 |
|  | 0.70 | 0.999 | 3 | 0.821 | 2 | 19.837 | 3 | 14.841 | 3 | 11 |
|  | 0.75 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.80 | 1 | 4 | 1 | 8 | 0.008 | 20 | 0.006 | 20 | 52 |
|  | 0.85 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.90 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
| 100 | 0.55 | 1 | 4 | 1 | 8 | 0.866 | 13 | 0.449 | 15 | 40 |
|  | 0.60 | 1 | 4 | 1 | 8 | 0.189 | 17 | 0.125 | 17 | 46 |
|  | 0.65 | 1 | 4 | 0.91 | 4 | 12.665 | 4 | 9.261 | 4 | 16 |
|  | 0.70 | 0.998 | 1 | 0.905 | 3 | 26.799 | 1 | 14.897 | 2 | 7 |
|  | 0.75 | 1 | 4 | 1 | 8 | 0.002 | 21 | 0.001 | 21 | 54 |
|  | 0.80 | 1 | 4 | 1 | 8 | 0.324 | 16 | 0.259 | 16 | 44 |
|  | 0.85 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.90 | 1 | 4 | 1 | 8 | 0.132 | 19 | 0.066 | 19 | 50 |
| 150 | 0.55 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.60 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.65 | 0.998 | 1 | 0.756 | 1 | 24.692 | 2 | 19.281 | 1 | 5 |
|  | 0.70 | 1 | 4 | 0.988 | 7 | 6.819 | 8 | 4.499 | 8 | 27 |
|  | 0.75 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.80 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.85 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.90 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
| 200 | 0.55 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.60 | 1 | 4 | 1 | 8 | 5.068 | 9 | 3.338 | 9 | 30 |
|  | 0.65 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.70 | 1 | 4 | 1 | 8 | 0.861 | 14 | 0.497 | 13 | 39 |
|  | 0.75 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.80 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.85 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.90 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
| 250 | 0.55 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.60 | 1 | 4 | 0.931 | 5 | 11.52 | 5 | 7.256 | 5 | 19 |
|  | 0.65 | 1 | 4 | 1 | 8 | 3.704 | 11 | 2.066 | 12 | 35 |
|  | 0.70 | 1 | 4 | 1 | 8 | 8.729 | 7 | 4.653 | 7 | 26 |
|  | 0.75 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.80 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.85 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | **0.90** | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
| 300 | 0.55 | 1 | 4 | 0.985 | 6 | 4.764 | 10 | 2.587 | 10 | 30 |
|  | 0.60 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.65 | 1 | 4 | 1 | 8 | 0.842 | 15 | 0.472 | 14 | 41 |
|  | 0.70 | 1 | 4 | 1 | 8 | 9.187 | 6 | 4.886 | 6 | 24 |
|  | 0.75 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.80 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.85 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |
|  | 0.90 | 1 | 4 | 1 | 8 | 0 | 22 | 0 | 22 | 56 |

**Table S-2** Performance of MFO-GB models on the test set

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Pop** | **Tr.atio** | **R2** | **Score** | **A10** | **Score** | **RMSE** | **Score** | **MAE** | **Score** | **Total** |
| 50 | 0.55 | -0.151 | 1 | 0.218 | 4 | 735.664 | 1 | 254.487 | 1 | 7 |
|  | 0.60 | 0.467 | 27 | 0.388 | 34 | 524.409 | 24 | 188.143 | 26 | 111 |
|  | 0.65 | 0.46 | 25 | 0.465 | 42 | 554.781 | 20 | 197.727 | 22 | 109 |
|  | 0.70 | 0.434 | 18 | 0.378 | 30 | 601.492 | 8 | 207.284 | 18 | 74 |
|  | 0.75 | 0.432 | 15 | 0.419 | 36 | 592.812 | 11 | 185.597 | 29 | 91 |
|  | **0.80** | **0.963** | **48** | **0.48** | **45** | **140.724** | **48** | **79.073** | **48** | **189** |
|  | 0.85 | 0.931 | 38 | 0.474 | 44 | 216.539 | 41 | 102.373 | 40 | 163 |
|  | 0.90 | 0.943 | 45 | 0.308 | 15 | 226.232 | 39 | 115.301 | 36 | 135 |
| 100 | 0.55 | 0.423 | 13 | 0.236 | 8 | 521.046 | 25 | 202.164 | 20 | 66 |
|  | 0.60 | 0.488 | 28 | 0.122 | 1 | 514.317 | 26 | 224.143 | 6 | 61 |
|  | 0.65 | 0.392 | 6 | 0.302 | 13 | 588.49 | 12 | 213.282 | 13 | 44 |
|  | 0.70 | 0.428 | 14 | 0.378 | 30 | 604.733 | 6 | 218.127 | 9 | 59 |
|  | 0.75 | 0.396 | 8 | 0.452 | 40 | 611.319 | 5 | 207.26 | 19 | 72 |
|  | 0.80 | 0.933 | 39 | 0.56 | 47 | 188.662 | 45 | 82.216 | 46 | 177 |
|  | 0.85 | 0.925 | 35 | 0.368 | 25 | 225.165 | 40 | 106.703 | 39 | 139 |
|  | 0.90 | 0.934 | 41 | 0.231 | 5 | 243.499 | 32 | 134.751 | 34 | 112 |
| 150 | 0.55 | 0.347 | 3 | 0.182 | 3 | 554.187 | 21 | 227.63 | 2 | 29 |
|  | 0.60 | 0.489 | 29 | 0.286 | 11 | 513.616 | 27 | 197.537 | 23 | 90 |
|  | 0.65 | 0.433 | 16 | 0.372 | 28 | 568.221 | 19 | 195.393 | 24 | 87 |
|  | 0.70 | 0.444 | 19 | 0.378 | 30 | 596.518 | 10 | 215.974 | 10 | 69 |
|  | 0.75 | 0.378 | 4 | 0.323 | 20 | 620.673 | 2 | 214.309 | 11 | 37 |
|  | 0.80 | 0.923 | 32 | 0.32 | 18 | 202.543 | 43 | 96.37 | 43 | 136 |
|  | 0.85 | 0.938 | 42 | 0.526 | 46 | 204.626 | 42 | 95.418 | 44 | 174 |
|  | 0.90 | 0.941 | 43 | 0.231 | 5 | 231.438 | 34 | 135.458 | 33 | 115 |
| 200 | 0.55 | 0.296 | 2 | 0.236 | 8 | 575.311 | 18 | 226.03 | 3 | 31 |
|  | 0.60 | 0.459 | 23 | 0.327 | 23 | 528.404 | 23 | 188.859 | 25 | 94 |
|  | 0.65 | 0.415 | 11 | 0.465 | 42 | 577.384 | 17 | 210.527 | 16 | 86 |
|  | 0.70 | 0.433 | 16 | 0.324 | 22 | 602.034 | 7 | 220.456 | 7 | 52 |
|  | 0.75 | 0.417 | 12 | 0.452 | 40 | 600.69 | 9 | 214.166 | 12 | 73 |
|  | 0.80 | 0.953 | 47 | 0.56 | 47 | 158.724 | 47 | 79.716 | 47 | 188 |
|  | 0.85 | 0.92 | 31 | 0.368 | 25 | 232.772 | 33 | 112.044 | 37 | 126 |
|  | 0.90 | 0.927 | 37 | 0.231 | 5 | 256.557 | 31 | 150.544 | 31 | 104 |
| 250 | 0.55 | 0.457 | 22 | 0.273 | 10 | 505.309 | 30 | 211.515 | 15 | 77 |
|  | 0.60 | 0.444 | 19 | 0.408 | 35 | 535.983 | 22 | 187.607 | 27 | 103 |
|  | 0.65 | 0.397 | 9 | 0.372 | 28 | 586.157 | 15 | 201.095 | 21 | 73 |
|  | 0.70 | 0.461 | 26 | 0.378 | 30 | 587.313 | 14 | 225.924 | 4 | 74 |
|  | 0.75 | 0.382 | 5 | 0.323 | 20 | 618.675 | 3 | 219.077 | 8 | 36 |
|  | 0.80 | 0.925 | 35 | 0.32 | 18 | 200.468 | 44 | 99.619 | 42 | 139 |
|  | 0.85 | 0.923 | 32 | 0.421 | 38 | 229.045 | 36 | 110.12 | 38 | 144 |
|  | 0.90 | 0.943 | 45 | 0.154 | 2 | 226.334 | 38 | 138.866 | 32 | 117 |
| 300 | 0.55 | 0.452 | 21 | 0.309 | 17 | 507.715 | 29 | 179.725 | 30 | 97 |
|  | 0.60 | 0.491 | 30 | 0.347 | 24 | 512.937 | 28 | 187.527 | 28 | 110 |
|  | 0.65 | 0.403 | 10 | 0.302 | 13 | 583.082 | 16 | 213.154 | 14 | 53 |
|  | 0.70 | 0.459 | 23 | 0.297 | 12 | 588.055 | 13 | 224.867 | 5 | 53 |
|  | 0.75 | 0.395 | 7 | 0.419 | 36 | 611.954 | 4 | 207.949 | 17 | 64 |
|  | 0.80 | 0.933 | 39 | 0.44 | 39 | 188.644 | 46 | 89.294 | 45 | 169 |
|  | 0.85 | 0.924 | 34 | 0.368 | 25 | 226.648 | 37 | 101.797 | 41 | 137 |
|  | 0.90 | 0.941 | 43 | 0.308 | 15 | 231.189 | 35 | 132.413 | 35 | 128 |

**2. Results of failure modes identification**

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**Fig. S-7.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 50.

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**Fig. S-8.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 100.

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A picture containing screenshot, colorfulness, graphics, magenta

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**Fig. S-9.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 150.

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**Fig. S-10.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 200.

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**Fig. S-11.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 250.

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**Fig. S-12.** Effect of training-test ratios on the MFO-GB model’s performance with population size of 300.

**Table S-3** Performance of MFO-GB models on the training set

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Pop** | **Tr.atio** | **Acc** | **Score** | **Pre** | **Score** | **Re** | **Score** | **f1** | **Score** | **Total** |
| 50 | 0.55 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.65 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.70 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.75 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.80 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | **0.85** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **4** |
|  | 0.90 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
| 100 | 0.55 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.65 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.70 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.75 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.80 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.85 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.90 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
| 150 | 0.55 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.65 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.70 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.75 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.80 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.85 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.90 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
| 200 | 0.55 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.65 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.70 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.75 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.80 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.85 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.90 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
| 250 | 0.55 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.65 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.70 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.75 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.80 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.85 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.90 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
| 300 | 0.55 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.60 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.65 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.70 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.75 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.80 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.85 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |
|  | 0.90 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 |

**Table S-4** Performance of MFO-GB models on the test set

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Pop** | **Tr.atio** | **Acc** | **Score** | **Pre** | **Score** | **Re** | **Score** | **f1** | **Score** | **Total** |
| 50 | 0.55 | 0.88 | 24 | 0.884 | 24 | 0.88 | 24 | 0.879 | 24 | 96 |
|  | 0.60 | 0.851 | 3 | 0.856 | 3 | 0.851 | 3 | 0.851 | 4 | 13 |
|  | 0.65 | 0.862 | 11 | 0.866 | 14 | 0.862 | 11 | 0.86 | 12 | 48 |
|  | 0.70 | 0.855 | 6 | 0.864 | 11 | 0.855 | 6 | 0.854 | 8 | 31 |
|  | 0.75 | 0.891 | 26 | 0.895 | 34 | 0.891 | 26 | 0.891 | 27 | 113 |
|  | 0.80 | 0.865 | 14 | 0.866 | 13 | 0.865 | 14 | 0.864 | 15 | 56 |
|  | **0.85** | **0.929** | **43** | **0.929** | **43** | **0.929** | **43** | **0.929** | **43** | **172** |
|  | 0.90 | 0.895 | 36 | 0.895 | 28 | 0.895 | 36 | 0.895 | 36 | 136 |
| 100 | 0.55 | 0.88 | 24 | 0.884 | 24 | 0.88 | 24 | 0.879 | 24 | 96 |
|  | 0.60 | 0.865 | 14 | 0.867 | 15 | 0.865 | 14 | 0.865 | 16 | 59 |
|  | 0.65 | 0.862 | 11 | 0.864 | 10 | 0.862 | 11 | 0.86 | 13 | 45 |
|  | 0.70 | 0.873 | 22 | 0.878 | 22 | 0.873 | 22 | 0.872 | 22 | 88 |
|  | 0.75 | 0.913 | 41 | 0.922 | 42 | 0.913 | 41 | 0.913 | 42 | 166 |
|  | 0.80 | 0.892 | 29 | 0.905 | 37 | 0.892 | 29 | 0.892 | 32 | 127 |
|  | 0.85 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 172 |
|  | 0.90 | 0.895 | 36 | 0.895 | 28 | 0.895 | 36 | 0.895 | 36 | 136 |
| 150 | 0.55 | 0.867 | 17 | 0.874 | 19 | 0.867 | 17 | 0.867 | 19 | 72 |
|  | 0.60 | 0.865 | 14 | 0.871 | 18 | 0.865 | 14 | 0.864 | 14 | 60 |
|  | 0.65 | 0.892 | 34 | 0.896 | 35 | 0.892 | 34 | 0.892 | 30 | 133 |
|  | 0.70 | 0.855 | 6 | 0.864 | 11 | 0.855 | 6 | 0.854 | 8 | 31 |
|  | 0.75 | 0.891 | 26 | 0.891 | 26 | 0.891 | 26 | 0.89 | 26 | 104 |
|  | 0.80 | 0.892 | 29 | 0.905 | 37 | 0.892 | 29 | 0.892 | 32 | 127 |
|  | 0.85 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 172 |
|  | 0.90 | 0.842 | 2 | 0.844 | 2 | 0.842 | 2 | 0.839 | 2 | 8 |
| 200 | 0.55 | 0.867 | 17 | 0.876 | 20 | 0.867 | 17 | 0.865 | 17 | 71 |
|  | 0.60 | 0.824 | 1 | 0.836 | 1 | 0.824 | 1 | 0.823 | 1 | 4 |
|  | 0.65 | 0.877 | 23 | 0.88 | 23 | 0.877 | 23 | 0.876 | 23 | 92 |
|  | 0.70 | 0.855 | 6 | 0.86 | 7 | 0.855 | 6 | 0.853 | 6 | 25 |
|  | 0.75 | 0.87 | 20 | 0.871 | 17 | 0.87 | 20 | 0.869 | 20 | 77 |
|  | 0.80 | 0.892 | 29 | 0.905 | 37 | 0.892 | 29 | 0.892 | 32 | 127 |
|  | 0.85 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 172 |
|  | 0.90 | 0.895 | 36 | 0.895 | 28 | 0.895 | 36 | 0.895 | 36 | 136 |
| 250 | 0.55 | 0.867 | 17 | 0.877 | 21 | 0.867 | 17 | 0.866 | 18 | 73 |
|  | 0.60 | 0.851 | 3 | 0.859 | 6 | 0.851 | 3 | 0.852 | 5 | 17 |
|  | 0.65 | 0.862 | 11 | 0.863 | 9 | 0.862 | 11 | 0.86 | 11 | 42 |
|  | 0.70 | 0.855 | 6 | 0.86 | 7 | 0.855 | 6 | 0.853 | 6 | 25 |
|  | 0.75 | 0.913 | 41 | 0.915 | 41 | 0.913 | 41 | 0.912 | 41 | 164 |
|  | 0.80 | 0.892 | 29 | 0.905 | 37 | 0.892 | 29 | 0.892 | 32 | 127 |
|  | 0.85 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 172 |
|  | 0.90 | 0.895 | 36 | 0.895 | 28 | 0.895 | 36 | 0.895 | 36 | 136 |
| 300 | 0.55 | 0.892 | 28 | 0.893 | 27 | 0.892 | 28 | 0.891 | 29 | 112 |
|  | 0.60 | 0.851 | 3 | 0.858 | 4 | 0.851 | 3 | 0.85 | 3 | 13 |
|  | 0.65 | 0.892 | 34 | 0.896 | 35 | 0.892 | 34 | 0.892 | 30 | 133 |
|  | 0.70 | 0.855 | 6 | 0.859 | 5 | 0.855 | 6 | 0.855 | 10 | 27 |
|  | 0.75 | 0.87 | 20 | 0.87 | 16 | 0.87 | 20 | 0.87 | 21 | 77 |
|  | 0.80 | 0.892 | 29 | 0.895 | 33 | 0.892 | 29 | 0.891 | 28 | 119 |
|  | 0.85 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 0.929 | 43 | 172 |
|  | 0.90 | 0.895 | 36 | 0.895 | 28 | 0.895 | 36 | 0.895 | 36 | 136 |

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Description automatically generated with low confidenceA screenshot of a computer

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**Fig. S-13.** Performance of MFO-GB models.